CSE 260M / ESE 260
Intro. To Digital Logic & Computer Design

Bill Siever
&
Michael Hall

Review & Expansion

Chapter 1 Sections

3. **Digital Abstraction**
4. **Number Systems**
5. Logic Gates

Abstraction

- Digital discipline
  - Discrete values: 0 and 1
    - Smallest unit of information: a binary digit. Also-know-as a *Bit*
  - (Mostly) Starting at gate level

Counting

Decimal	Binary
00	000
01	001
02	010
03	011
04	100
05	101
06	110
07	111

Binary Basics: Number Line

Decimal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111

Conversions: Place Value

Place Value: Base 2
To *Decimal*
Example: 110_2 (or 3'b110)

Digits	1	1	0
Place Value (Decimal)	4	2	1
Place Value	2^2		
Expansion	1×2^2	$+1\times2^1$	$+0\times2^0$

Digit in decimal

Value of place in decimal

Sum them all

Hexadecimal

- Convenient, compact way to deal with binary
- Each hex digit = exactly 4 binary digits
- Sixteen digits:
  0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Counting

Decimal	Binary	Hex
00	0000	
01	0001	
02	0010	
03	0011	
04	0100	
05	0101	
06	0110	
07	0111	
08	1000	
09	1001	
10	1010	
11	1011	
12	1100	
13	1101	
14	1110	
15	1111	

Counting

Decimal	Binary	Hex
00	0000	0
01	0001	1
02	0010	2
03	0011	3
04	0100	4
05	0101	5
06	0110	6
07	0111	7
08	1000	8
09	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Hex Notations

- Programming: 0x followed by digits, like 0xAB7
- Verilog: *n*'h*digits*

Hex to Binary

- What is 12’hAB7 in binary?

Hex to Decimal

- What is 12'hAB7 in decimal?
  - Option 1: Convert to binary and then the binary to decimal
  - Option 2: Use place-value approach

Hex to Decimal

- What is 12’hAB7 in decimal?

What is 123 in hex?

- Greedy approach
  - A lot like binary, but have to ask “how many times does this go in…”

Place Value (Decimal)	256	16	1
Place Value In terms of Base	16^2	16^1	16^0

Why hex?

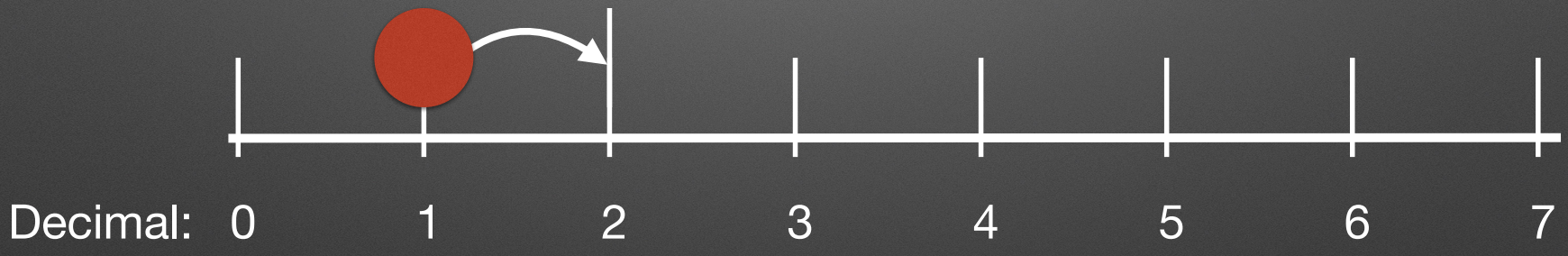
- Consider 32-bit numbers
  - 0xA764 1CD8
  - Vs. 1010 0111 0110 0100 0001 1100 1101 1000

Review & Expansion: Arithmetic

Decimal Addition: Bunch of Rules

Rules just “encode” moving right on the number line

Ex: 1+2



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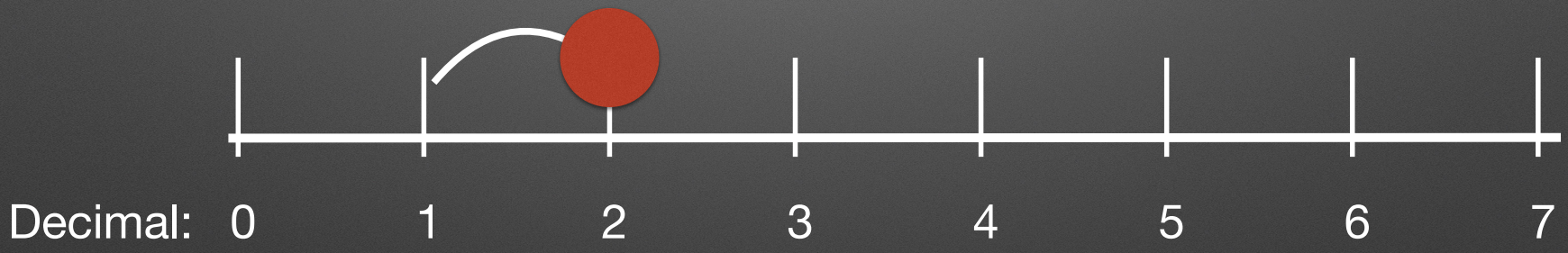

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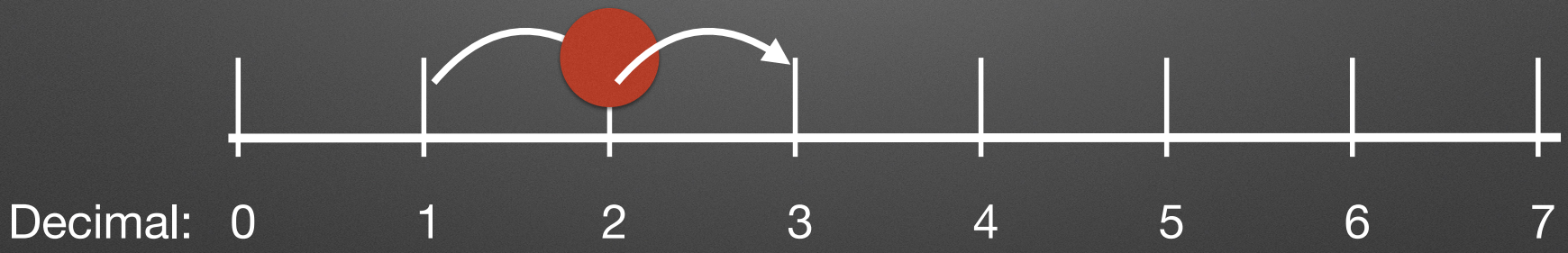

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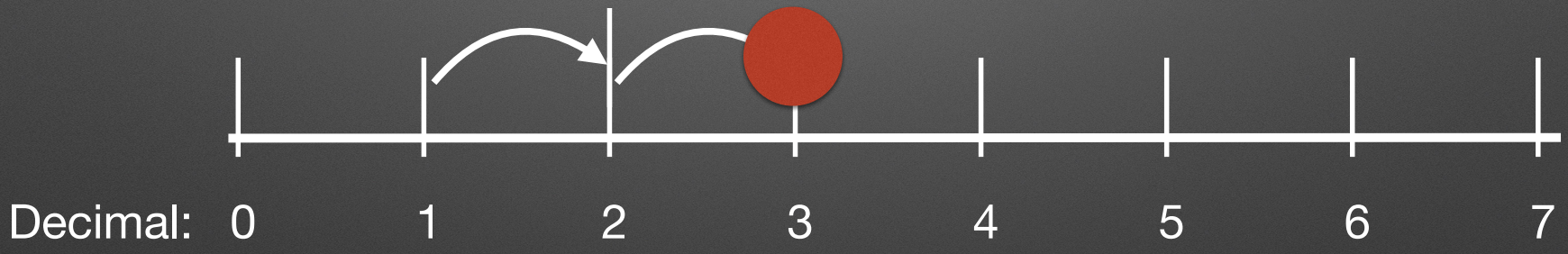


Binary Addition Rules

- Condensed
  - No ones: 0+0+0 = 00
  - One one: 0+0+1 = 01
  - Two Ones: 0+1+1 = 10
  - Three Ones: 1+1+1 =11

Binary Addition Rules: Fully Elaborated

0+ 0+ 0	=	00
0+ 0+ 1	=	01
0+ 1+ 0	=	01
0+ 1+ 1	=	10
1+ 0+ 0	=	01
1+ 0+ 1	=	10
1+ 1+ 0	=	10
1+ 1+ 1	=	11

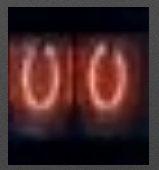
Review Problem

- Add 4’b0110 + 4’b0010

Fixed Width Arithmetic

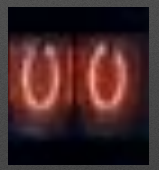
Fixed Width Arithmetic

Fixed Width Arithmetic



Fixed Width Arithmetic

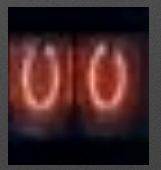
- Consider the following problems:
  - 1 + 2 =
  - 95 + 6 =
  - 80 + 20 =



Fixed Width Arithmetic

- Consider the following problems:
  - 01 + 02 = 03 => 03
  - 95 + 06 = 101 => 01
  - 80 + 20 = 100 => 00

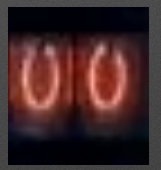
- Consider the following problems:
  - 03 op number = 03
  - 101 op number = 01
  - 100 op number = 00



Fixed Width Arithmetic

- Consider the following problems:
  - 01 + 02 = 03 => 03
  - 95 + 06 = 101 => 01
  - 80 + 20 = 100 => 00

- Consider the following problems:
  - 03 % 100 = 03
  - 101 % 100 = 01
  - 100 % 100 = 00



Fixed Width Arithmetic

- $A + B$ With n-digit inputs and result in base b
- Result is $(A + B) \% b^n$

Modular Arithmetic & The Number Line (Binary, 3-bit)

Decimal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111

What's 1+2?

Modular Arithmetic & The Number Line (Binary, 3-bit)

Decimal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111

What's 6+2?

Full, binary number line (2^n values): Exceeding End Wraps!

Decimal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111

What's 6+2?

Going Negative

- What about negative numbers?
  - Easy option: Encode the concept of a sign
    - 0 = positive
    - 1 = negative
  - Pick a bit to represent the sign

Example: 4-bit sign / magnitude

- Format

Place:	Sign	2^2	2^1	2^0

  - Sign first (common choice)
- What is the decimal value of the 4-bit, sign-magnitude
  - 4’b1001
  - 4’b0110

Really Negative

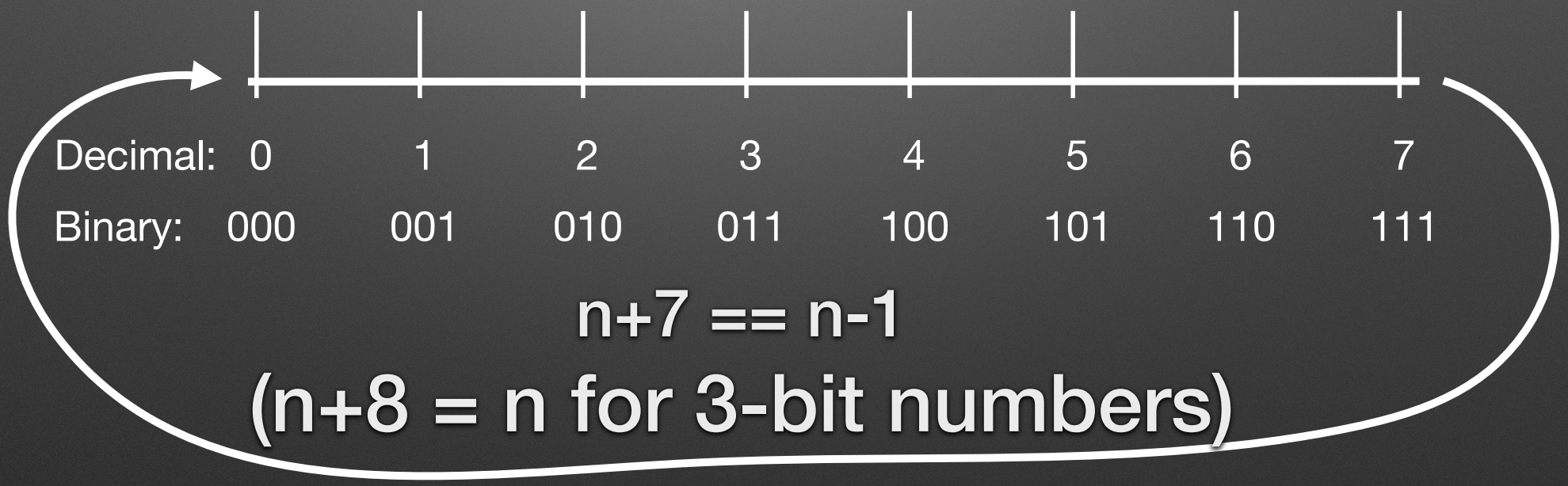
- What is the decimal value of the 4-bit, sign-magnitude
  - 4’b1000
  - 4’b0000
  - Weird
- What is 4’b1001 + 4’b0110?
  - Arithmetic is messy with sign/magnitude!

UNSIGNED Again…

Challenge: Describe the result of n+7?

Decimal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111

Challenge: Describe the result of n+7?



n+7 == n-1

(n+8 = n for 3-bit numbers)

Challenge: How can you emulate n-2?

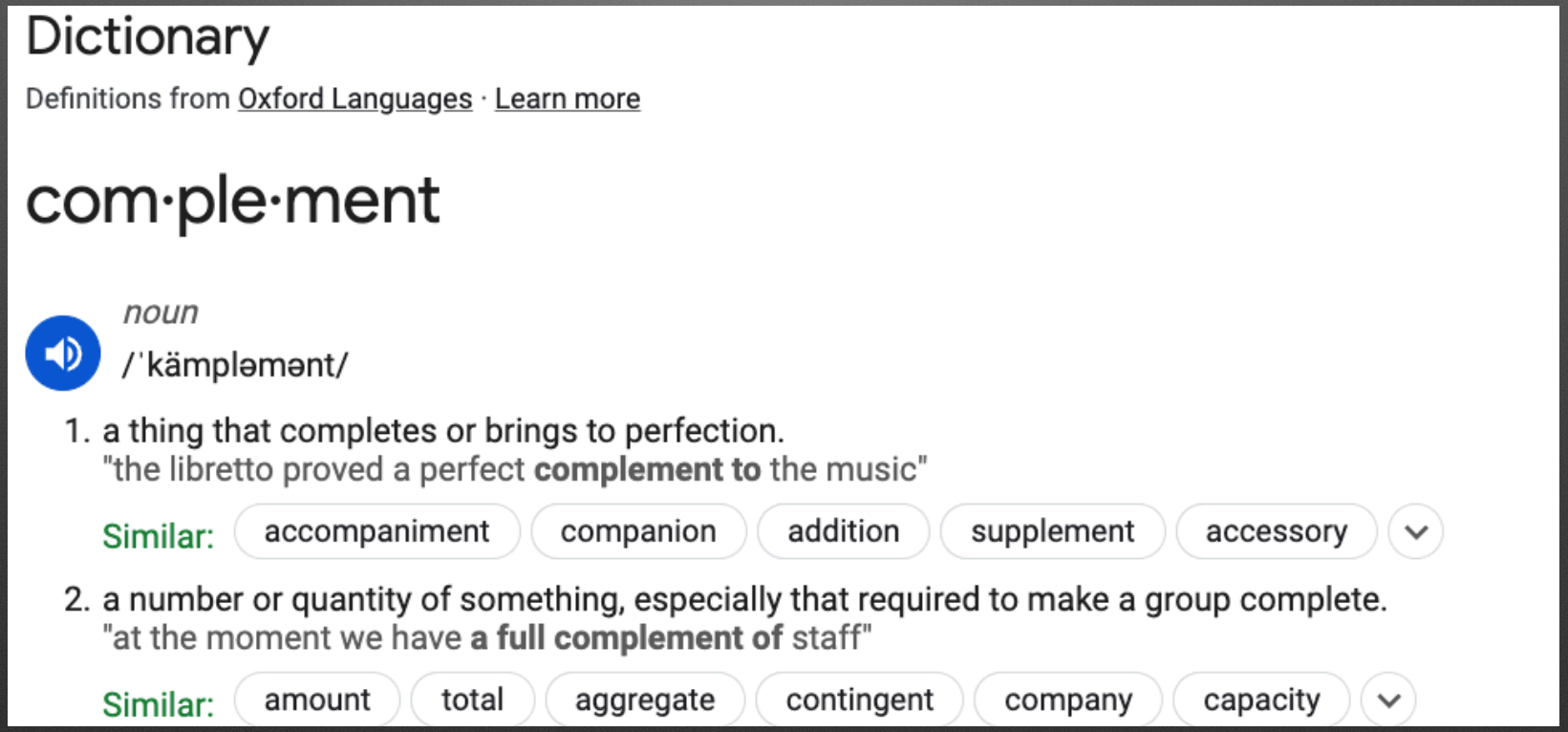
Decimal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111

The Magic of Fixed Width numbers (modular arithmetic): Addition can emulate subtraction!

Decimal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111
2's comp behavior:					-4	—3	-2	-1

Two's Complement Sign Representation

The “Two’s Complement”



Dictionary

Definitions from Oxford Languages · Learn more

com·ple·ment

noun

/ˈkämpləmənt/

1. a thing that completes or brings to perfection.
   "the libretto proved a perfect **complement to** the music"

   Similar: accompaniment companion addition supplement accessory

2. a number or quantity of something, especially that required to make a group complete.
   "at the moment we have **a full complement of** staff"

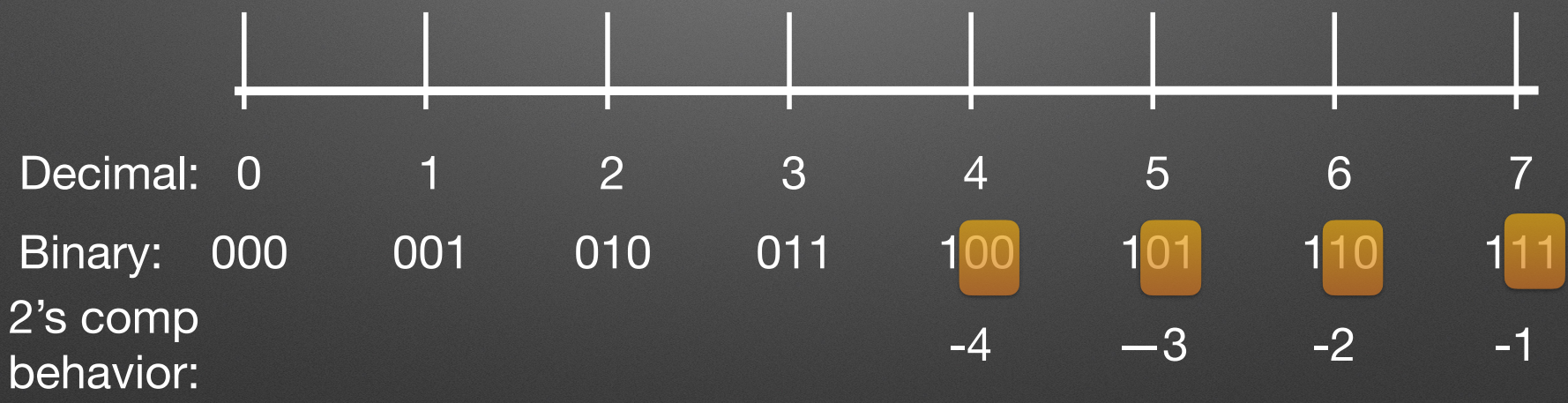
   Similar: amount total aggregate contingent company capacity

Dividing the Line

- It'll be nice if we have about half as many negatives as positives. We'll split the line in half, like we've already done.
  - How can we identify if a binary number is positive or negative?
    - Ex: 010? Or 110?

Decimal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111
2's comp behavior:					-4	—3	-2	-1

What about conversions & place/value stuff?



Consider the Upper Bit to be Negative

Place Value (Decimal)	-4	2	1
Place Value In terms of Base	$-(2^2)$	2^1	2^0

Consider the Upper Bit to be Negative

Place Value (Decimal)	-4	2	1
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What is the decimal value of the 3-bit, 2's complement numbers:

110

011

Consider the Upper Bit to be Negative

Place Value (Decimal)	-4	2	1
Place Value In terms of Base	$-(2^2)$	2^1	2^0

What is the 3-bit, 2's complement representation of:

2

-4

-5

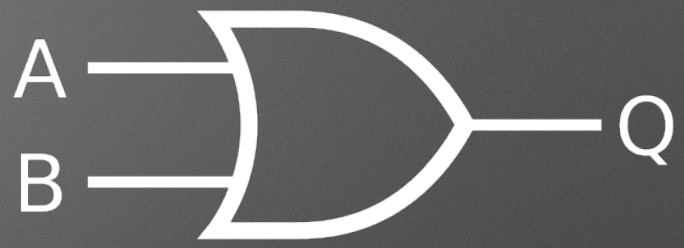
4-bit Two’s Complement

- What is the decimal value of the 4-bit, Two’s Complement Number
  - 0xA
  - 0x7

Gates & Tables

- We can describe the *final behavior* simple machines, like gates, with tables
  - Truth table
    - Row for each possible combination of inputs
      (Binary; Fixed width counting)
    - Column for each input and each output

And Gate

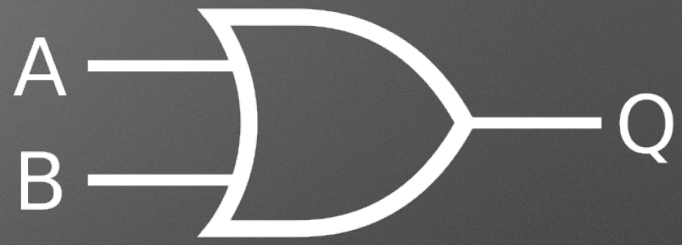


- How many rows in truth table?
- How many columns?

OR: Gate…A real machine

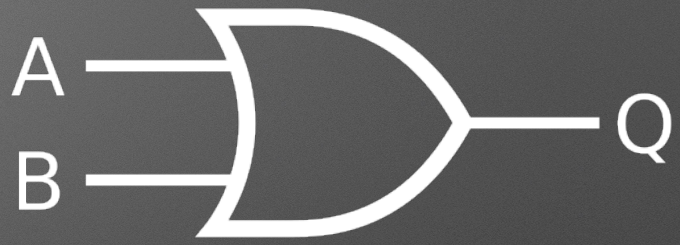
- Spintronics game: https://upperstory.com/en/spintronics/

New Idea: Behavior OVER TIME



- Gate represents a machine of some sort machine
  - in the real world
- They are not instantaneous

The timing diagram



Intro to JLS

Coming Due Dates

- Prep 1 (Canvas): ASAP! No later than Jan 23 @ 11:59pm
- Homework 1: Sunday by 11:59pm / Gradescope
  - Based on chapter 1
- Prep 1: Monday @ 11:59pm (accepted until Jan 23 @ 11:59pm
- Future preps not accepted late.

Questions

- Sign stuff?
  - Sign extension?
- Conversions
  - Any tricks?
- Memorize binary for exam?
  - Some
- Units in binary? Kibi?